Turbulent Boundary Layer on a Cylinder in Axial Flow

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Abstract

RESEARCH on the turbulent boundary layer that develops on a cylinder in axial flow is reviewed in the report accompanying this synoptic. Experimental results indicate that the transverse curvature results in a higher coefficient of friction and a "fuller" velocity profile than for a planar boundary layer. However, appropriate scaling laws and nondimensional scaling parameters have been elusive. The few turbulence measurements such as Reynolds stress and intermittency that are available for a cylindrical boundary layer suggest that the distribution of turbulent quantities in the boundary layer is somewhat different from a planar boundary layer, particularly as the boundary layer becomes thick compared to the radius of the cylinder. This is most likely a result of the tendency for a cylindrical boundary layer to become wake-like as the cylinder becomes very small. Measurements of turbulence intensity and detection of turbulence-generating events in a cylindrical boundary layer suggest that the mechanism for the production of turbulence near the wall is similar to that for other wall-bounded flows. However, there is experimental evidence that the outer flow interacts with the near-wall flow to modify the mechanism for the generation of turbulence.

Contents

The effect of transverse curvature on the turbulent boundary layer that develops as a fluid flows parallel to a cylindrical surface has applications to boundary layers on many different objects including missiles, aircraft, ships, torpedoes, towed submerged cables or cylindrical bodies, and glass or polymer fibers during fabrication. The report accompanying this synoptic reviews the research on cylindrical boundary layers in the past 40 years.

Like the planar boundary layer, the cylindrical boundary layer is two dimensional (in the streamwise and wall-normal directions). But a cylindrical boundary layer is not as simple as its two-dimensional character implies because of the existence of an additional length scale, the radius of transverse curvature a. The integration of this additional length scale into a nondimensional parameter leads to several possibilities†: $a_+ = aU_\tau/\nu$, δ/a, $R_a = aU_\infty/\nu$, and $\xi = \sqrt{(\nu x/U_\infty a^2)}$, where U_τ is the friction velocity, ν is the kinematic viscosity, δ is the boundary-layer thickness, U_∞ is the outer freestream velocity, and x is the streamwise coordinate. The range of experimental data available spans three or four orders of magnitude for each of these nondimensional parameters, but the variation in the parameters is primarily a result of changing the radius of transverse curvature or the freestream velocity. Very little data are available based on parameter variation accomplished by measuring the boundary-layer characteristics at different axial locations given the same transverse curvature in order to vary a_+, δ/a, or ξ independently from R_a. This deficiency has made the selection of an appropriate nondimensional parameter difficult.

Clearly, R_a is the least appropriate nondimensional parameter, since it omits any effect due to varying wall shear stresses or integral thicknesses at different axial positions. For this reason, a_+ or δ/a are logical nondimensional parameters except that they suffer from the difficulty that detailed measurements of the boundary must be made to determine these parameters. The parameter ξ overcomes this deficiency, although the employment of this parameter has been minimal. Thus, the appropriate nondimensional parameter that incorporates the effect of transverse curvature is still debatable. However, the transverse curvature ratio δ/a is preferred in this report because of its clear geometric interpretation.

Mean Velocity Profile

The mean velocity profile of a cylindrical boundary layer is substantially different from that of a planar boundary layer as δ/a becomes large. The mean velocity profile is fuller than in the planar case.[1] When plotted in the usual wall coordinates, a logarithmic region is retained although the slope of the velocity profile in the log region depends upon the transverse curvature. For small transverse curvature [$\delta/a < \mathcal{O}(1)$], the log law appears identical to the planar log law with von Kármán's constant, $\kappa = 0.4$, describing the slope of the logarithmic region. However, as the transverse curvature increases [$\delta/a > \mathcal{O}(1)$], κ grows as a linear function of δ/a.[2]

Unfortunately, even the use of the usual wall coordinates for scaling the distance from the wall can be questioned because the effect of the transverse curvature is not directly taken into account. This has led to an alternative nondimensional form for the wall-normal coordinate based upon the mean velocity profile in the viscous sublayer. In this region, it can be shown that $U_+ = a_+ \ln[1 + (y/a)]$. Since this scaling is appropriate in the sublayer, it was suggested that $\{a_+ \ln[1 + (y/a)]\}$ be substituted for y_+ in the planar log law. Although this scaling for the wall-normal coordinate reflects the geometry of the cylindrical boundary layer, considerable controversy surrounded its introduction, because it implies that the sublayer and the log region respond similarly to transverse curvature.

A weakness in the presentation and analysis of nearly all velocity data measured for cylindrical boundary layers is that the scaling depends upon the accurate measurement of the wall shear stress. Attempts have been made to measure the skin friction or drag coefficient in a cylindrical boundary layer using many methods including floating elements, towing experiments, and, under certain circumstances, Preston tubes. In general, the measured coefficient of friction has been higher than that for a planar boundary layer at the same momentum-thickness Reynolds number. But the difficulty in measuring the wall shear stress makes the published velocity data subject to question. As a result, proper scaling and similarity laws have been difficult to determine for cylindrical boundary layers.

Nearly a dozen proposals for similarity laws for the logarithmic region of a cylindrical boundary layer have been made. Many of these similarity laws are based on planar mixing lengths or variations of the planar mixing length using the alternative wall-normal coordinate, $\{a_+ \ln[1 + (y/a)]\}$, to ac-

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†All variables subscripted with "+" have been nondimensionalized using ν and $U_\tau = \sqrt{(\tau_w/\rho)}$, where τ_w is the wall shear stress and ρ is the fluid density.

count for transverse curvature. Some of the proposed similarity laws seem to match the mean velocity data better than others, but no single similarity law stands out compared to the others.

In its outer region, the mean velocity profile of a cylindrical boundary layer deviates from a logarithmic velocity profile by falling below the logarithmic profile. This has been termed a "negative" wake. But the negative wake of a cylindrical boundary layer is less obvious than the positive wake in a planar boundary layer. Consequently, the existence of the wake region in a sense similar to a planar boundary layer has been questioned and is unresolved.

Many analytic models of a cylindrical boundary layer have been proposed. Early work concentrated on power-law formulations of the velocity profile based on planar boundary layers. Other early models were based on planar mixing-length arguments. Several models used planar mean velocity profiles with corrections for transverse curvature. The results of some models correspond qualitatively to experimental results by predicting that skin friction increases as the transverse curvature increases.

Turbulence in Cylindrical Boundary Layers

When the transverse curvature is small [$\delta/a < \mathcal{O}(1)$], the measurements of the Reynolds stress, turbulence intensity, and wall pressure fluctuations appear similar to other planar wall-bounded flows.

For larger transverse curvature [$\delta/a > \mathcal{O}(1)$], experimental results are limited, but provide some insight into the effect of transverse curvature on the turbulence field. Consider three turbulence measurements near the wall which are most likely closely related to the mechanism for the generation of turbulence. First, turbulence event detection schemes such as variable interval time averaging (VITA) and uv-quadrant detection for cylindrical boundary layers at moderate transverse curvature [$\delta/a = \mathcal{O}(8)$] provide conditionally averaged events that are similar to those for planar boundary layers.[4] Second, the turbulence intensity near the wall has nearly the same maximum value and occurs at the same distance from the wall in both the streamwise and wall-normal directions as in other wall-bounded flows.[4] Finally, the convection velocity of wall pressure fluctuations is nearly the same as that for the planar boundary layers.[5] These results suggest that the turbulence-generation mechanism in a cylindrical boundary layer is related to the burst cycle.

In spite of several similarities in turbulence measurements near the wall between a cylindrical boundary layer with other wall-bounded flows, some interesting differences result from moderate transverse curvature of the wall [$\delta/a = \mathcal{O}(4)$]. For instance, the wall pressure spectrum for a cylindrical boundary layer contains a greater energy density at higher frequencies than that of a planar boundary layer.[5] Another difference is in the measured contours of constant wall pressure correlation. These contours are nearly circular in the streamwise-spanwise plane for a cylindrical boundary layer instead of being elongated in the spanwise direction as in a planar boundary layer. The Reynolds stress drops off quite quickly with distance from the wall compared to the planar case.[2] This is most likely a result of the increased spreading of the flowfield to larger and larger circumferences moving away from the cylinder compared to a flat wall where there can be no spreading.

Another striking difference between the boundary layer on a flat plate and the boundary layer on a cylinder is of the motion of large-scale structures in the outer portion of the boundary layer. In a cylindrical boundary layer, these structures can readily move from one side of the cylinder to the other side when the cylinder is small in comparison to the boundary-layer thickness [$\delta/a = \mathcal{O}(20)$].[4] This makes a cylindrical boundary layer more like an axisymmetric wake than like a boundary layer. Unlike a flat plate, the wall of a cylinder does little to constrain the motion of large-scale structures within the boundary layer. This effect is also evident in the measurements of intermittency where the mean location of the interface between turbulent and nonturbulent flow is farther from the wall in a cylindrical boundary layer than in a planar boundary layer.[4] The unconstrained motion of eddies allows them to "fill out" the boundary layer resulting in the interface being farther from the wall.

Perhaps the most interesting aspects of a cylindrical boundary layer is in the interaction between the boundary layer-like turbulence production near the wall of the cylinder and the wake-like character of the flow away from the immediate vicinity of the wall. It has been proposed that the pressure field from large eddies passing over the cylinder produce a massaging action in a localized region near the wall.[1] This random action promotes an unstable inflectional velocity profile near the wall resulting in a burst. Although the burst itself is very similar to a burst in other wall-bounded flows as evidenced by turbulent event-detection schemes, the outer wake-like portion of the boundary layer promotes the burst to occur. This theory is supported by considering the burst frequency as detected by the VITA technique.[4] In a planar boundary layer the burst frequency scales with inner variables, whereas in a cylindrical boundary layer the burst frequency scales with outer variables suggesting a stronger influence from the wake-like outer portion of the boundary layer.

It appears that as the boundary-layer thickness becomes large compared to the radius of curvature of a cylinder, the flow is a hybrid of an axisymmetric wake and a boundary layer. The wall acts to continuously convert mean flow energy into turbulent energy at the wall of the cylinder. Thus, the cylinder produces vorticity and turbulence near the centerline of the wake and acts to continuously regenerate the wake. But because the cylinder can be so small in comparison to the dimensions of the boundary layer, it can only have a slight influence on the outer portion of the boundary layer which becomes wake-like.

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